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1997 J. Phys.: Condens. Matter 9 5413

(http://iopscience.iop.org/0953-8984/9/25/009)

J. Phys.: Condens. Matter **9** (1997) 5413–5427. Printed in the UK PII: S0953-8984(97)81076-4

Conductivity of weakly insulating amorphous nickel–silicon films below the metal–insulator transition

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Received 20 January 1997, in final form 11 April 1997

Abstract. The electronic conductivity has been measured in homogeneous, weakly insulating, amorphous nickel–silicon films located just below the metal–insulator transition (MIT). The conductivity follows a simple CT^z power-law dependence with $z \approx 1/2$ over a large temperature interval. In contrast, a Mott variable-range hopping expression could not be fitted successfully through these zero-field conductivity data. The CT^z behaviour can be explained using the three-dimensional (3D) electron–electron interaction (EEI) theory. The negative magnetoconductance data observed in these weakly insulating films can be fitted nicely using only the 3D EEI theory. A crossover of the conductivity from the simply power-law CT^z dependence at high temperatures to an activated hopping-law dependence in the liquid helium temperature region is observed; this transition is attributed to changes in the energy dependence of the density of states near the Fermi level. The conductivity of these weakly insulating films can be fitted well over three decades of temperature using an empirical scaling expression suggested by Möbius *et al*.

1. Introduction

The approach to the metal–insulator transition (MIT) from the *metallic* side in two-dimensional (2D) systems has been studied experimentally by many groups [1–5]. A good summary of the experimental results on the metallic side of the MIT can be found in Belitz and Kirkpatrick's review article [6] and Castners review paper [7]. Surprisingly, there is very little work published on *insulating* three-dimensional (3D) films located just below the MIT. This might be due partially to the lack of theories that predict the 3D conductivity behaviour immediately below the MIT and partially due to the experimental constraint that the 3D film must be extremely homogeneous in composition throughout its thickness. For example, any inhomogeneous thin layer, 'rich' in metal content within the film, would 'short out' the remaining insulating regions of the film, causing the film to appear metallic. In contrast, if the metal were to be uniformly distributed throughout the film, the film would appear to be insulating. Thus, film preparation is critical in such a study.

Gantmakher *et al* [8] have presented an interesting argument suggesting that the conductivity just below the MIT has the functional dependence

$$\sigma(T) = a + bT^{1/3} \tag{1}$$

where $a < 0$ [8]. We believe that a negative value for 'a' is unphysical. However, we can generalize their argument to constrain 'a' to be zero; hence, one obtains the surprising prediction that the conductivity just below the MIT might follow a simple temperature

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power-law dependence, namely, that the resistivity should go as $\rho \propto T^{-z}$. The value for the exponent z of $1/3 < z < 1/2$ had originally been predicted by Finkel'shtein [9]; and Newson and Pepper later suggested that $z = 1/3$ just above the MIT [10].

None of the films in this paper are sufficiently insulating to display the Mott to Efros–Shklovskii (ES) variable-range hopping (VRH) conductivity transition that is often observed in *strongly insulating* films [11, 12]. Data for a strongly insulating a:Ni_xSi_{1-x} film are presented in [13].

For *weakly insulating* films, Möbius *et al* [14] have suggested a unique empirical scaling expression that is the *multiplicative decomposition* of two functions,

$$\sigma(T) = \xi_{low}(T, T_0, y)\xi_{hi}(T) \tag{2}$$

where $\xi_{low}(T, T_0, y)$ is a low-temperature VRH expression given by $\xi_{low} = 1/\exp(T_0/T)^y$, and $\xi_{hi}(T)$ is a function describing the high-temperature conductivity behaviour. T_0 is a characteristic temperature; and the exponent y would take on a value of $\frac{1}{4}$ for the case of 3D Mott VRH, a value of $\frac{1}{2}$ for Efros–Shklovskii VRH hopping and a value of 1 for nearest-neighbour hopping. Shortly, we will present experimental evidence that suggests that the 3D electron–electron interaction (EEI) theory describes the high-temperature conductivity data very well for these 'weakly insulating' films. Hence, we will associate $\xi_{hi}(T)$ with the 3D EEI expression, $\xi_{hi}(T) \approx CT^{1/2}$, and we propose the following Möbius scaling law expression

$$\sigma(T) = CT^{1/2}/\exp(T_0/T)^y \tag{3}$$

where the three fitting parameters, C, T_0 and y are evaluated from the data. Notice that in the high-temperature limit, $\sigma \rightarrow T^{1/2}$; and that in the low temperature limit, $\sigma \rightarrow \exp[-(T_0/T)^y]$.

Many studies have been made on thin 2D films [1–5]; interpretation of the metal–insulator transition is complicated in 2D systems by superconductivity and by the strength of the spin–orbit interaction that enters in the weak localization theory calculations. In the normal state, thin 2D films are expected to be insulating owing to the $\ln T$ dependence of the EEI contribution [15] and of the weak localization (WL) contribution to the conductivity [16]. We have avoided studying 2D films because of the complicating $\ln T$ dependence.

The amorphous Ni_xSi_{1-x} structure in the form of 3D films appears to be a good system to investigate near the MIT since the conductivity in 3D is not complicated by percolation, magnetic and granular effects and probably not by superconductivity [17]. Dammer observed in *highly insulating* a:Ni_xSi_{1-x} films an activated conductivity following a $\sigma(T) = \sigma_0/\exp(T_0/T)^y$ dependence with an exponent $y \approx 0.45$; the characteristic temperatures T_0 were in the range 500–6500 K [18].

2. Analysis techniques near the metal–insulator transition

Thin films may be classified as being either insulating or metallic. Insulating films are defined as exhibiting infinite resistivity (zero conductivity) at absolute zero in temperature. In contrast, metallic films always display a finite resistivity or a non-zero positive conductivity at absolute zero.

Strongly insulating films exhibit an activated hopping conductivity which can be described by the VRH expression

$$\sigma(T) = \sigma_0[\exp -(T_0/T)^y] \tag{4}$$

where σ_0 is the prefactor, T_0 is a characteristic temperature and y is an exponent.

In contrast, the conductivity of a 3D metallic film at sufficiently low temperatures can be described by a power-law expression

$$\sigma(T) = \sigma(0) + CT^z \tag{5}$$

where $\sigma(0)$ is the finite zero-temperature conductivity, C is a prefactor and z is the exponent of the temperature power law. Equation (5) might approximate the conductivity contributions of the 3D EEI theory [15] and/or of the 3D WL theory [16]. Note that in the above procedures, y and z are free fitting parameters.

A useful technique to identify the MIT was previously introduced in [19] and [20]. The mathematical function $w(T)$ exhibits distinctively different temperature behaviour for insulating and metallic films:

$$w(T) = \mathrm{d}\ln\sigma/\mathrm{d}\ln T = (T/\sigma)\mathrm{d}\sigma/\mathrm{d}T. \tag{6}$$

For strongly insulating films exhibiting VRH conductivity, inserting equation (4) into equation (6) yields

$$w(T) = y(T_0/T)^y. \tag{7}$$

Note that $w(T)$ *increases to infinity* as the temperature approaches absolute zero. By making a linear regression fit of the $\log[w(T)]$ versus $\log T$ data, one can directly extract values for the hopping exponent y and the characteristic temperature T_0 using equation (7). The slope of the straight line fit is equal to the exponent y and the intercept I of the fit is related to the characteristic temperature via $T_0 = (10^I/y)^{1/y}$.

For 3D metallic films exhibiting slowly decreasing conductivities with decreasing temperatures, equation (5) can be substituted into equation (6) to yield

$$w(T) = zCT^z/[\sigma(0) + CT^z] = zCT^z/\sigma(T). \tag{8}$$

Observe that if the film is indeed metallic and exhibits a finite positive conductivity $\sigma(0)$ at absolute zero, then $w(T)$ *should extrapolate to zero* at absolute zero. For these cases, linear regression fits of the $\log(w\sigma)$ versus $\log T$ data yield values for the exponent z and the prefactor C using equation (8). Values for $\sigma(0)$ follow from one conductivity data point for each different film. Extrapolation of $\sigma(0)$ to zero as a function of metal content will yield another estimation for the critical metallic content x_c at the MIT [20].

Samples might also exhibit *temperature independent* values of w. Such a temperature independent behaviour of w can be realized only if $\sigma(0)$ is set to zero in equation (8). For this case, the film is weakly insulating, since $\sigma(T) \to 0$ as $T \to 0$ K [21]; and the conductivity data can be describe using the simple temperature power-law expression

$$\sigma(T) = CT^z \tag{9}$$

with C and $z = w$ being the two fitting parameters. Note that a VRH law *cannot* be fitted successfully to conductivity data that exhibit w values which are temperature independent. We refer to films having conductivites described by equation (9) as 'weakly insulating' films [21].

3. Film preparation and characterization

Amorphous Ni_xSi_{1-x} films were prepared by co-evaporating Ni and Si using two electron guns. Fabrication and characterization details are given in [17]. During each evaporation, the rate of each source was monitored; if there were any noticeable deviations in either of the two rates, the series was discarded. More than ten attempts were made before the present homogeneous series was achieved; the homogeneity in the thickness of the films

is believed to be better than ±2.5%, based upon deviations from a linear regression fit of the evaporation rate data. Five out of the 23 films from this series were located on the insulating side just below the MIT.

The nickel content x in at.% Ni based upon EDAX measurements [22], the film thickness t and the room temperature conductivity σ_{RT} are listed in table 1 for the insulating a:Ni_xSi_{1-x} films.

Table 1. Nickel content x, thickness t and the room temperature conductivity σ_{RT} for the weakly insulating a:Ni_xSi_{1-x} films.

Film No, series 300	x (at.% Ni, EDAX)	t(Å)	$\sigma_{RT}(\Omega\ \mathrm{cm})^{-1}$
23	19.5	1210	42.3
22	20.3	1225	54.3
21	21.2	1235	73.0
20	22.3	1235	88.3
19	23.5	1230	116

4. Experimental results

Amongst the 23 homogeneous films obtained in series No 300, five of the films fell on the insulating side of the MIT. The metallic conductivity properties of the other 18 films are described in [17]. There is strong experimental evidence that these films are 3D owing to the $B^{1/2}$ dependence in their magnetoconductance data. The surprising experimental result was that the *metallic* conductivities at low temperatures could be well described using the empirical fit, $\sigma(T) = \sigma(0) + CT^z$, with $C \approx 4\ (\Omega\ \mathrm{cm})^{-1}$ and $z \approx 0.53$. These values for C and z agreed well with the EEI predictions that $C \approx 4\ (\Omega\ \mathrm{cm})^{-1}$ and $z = 1/2$ [15, 17]. The zero temperature conductivity term $\sigma(0)$ had a linear dependence upon the nickel content x, decreasing to zero as $\sigma(0) = \sigma_0(x - x_c)^1$. The critical Ni content x_c at the MIT fell between films Nos 19 and 18. Moreover, film No 18 exhibited w's which tended to zero as $T \to 0$, thus identifying this film as metallic, while film No 19 had w's which extrapolated to a *finite* value as $T \to 0$, making this film an insulator [17]. Thus, we identified film No 19 as the first *insulating* film.

Film No 21 best represents the behaviour of a 'weakly insulating' film located slightly below the MIT. A plot of $w = \mathrm{d}\ln\sigma/\mathrm{d}\ln T$ versus T in figure 1 reveals that w is *temperature independent* between 200 K down to 3 K and is equal to the average constant value of 0.50. A constant w implies that the conductivity can be fitted to a simple power-law expression $\sigma(T) = CT^z = 3.66T^{0.50}\ (\Omega\ \mathrm{cm})^{-1}$ as illustrated in figure 2 by the full curve; the empirical fit is good. Below 3 K, the w values slowly start to increase as seen in figure 1, suggesting the existence of a 'crossover' regime to activated hopping conductivity at much lower temperatures. Our lowest temperature of 0.45 K was not sufficient to define the VRH expression for this film.

It has been common practice amongst experimentalists to fit a Mott VRH law [11] through data taken on insulating samples located just below the MIT. In figure 2 we show a forced fit of the Mott law $\sigma_{Mott}(T) = \sigma_0/\exp(T_{Mott}/T)^{1/4} = 160/\exp(406/T)^{1/4}$ through the data of film No 21. Not only is the fit (broken curve) unacceptable, but also the criterion that the ratio of the optimum hopping distance r_{opt} to the localization length ξ be greater than one ($r_{opt}/\xi > 1$) fails over the entire temperature range [7]. For the data to be valid for the Mott law, recall that $r_{hop}/\xi = 0.375(T_{Mott}/T)^{1/4} > 1$. In our temperature interval

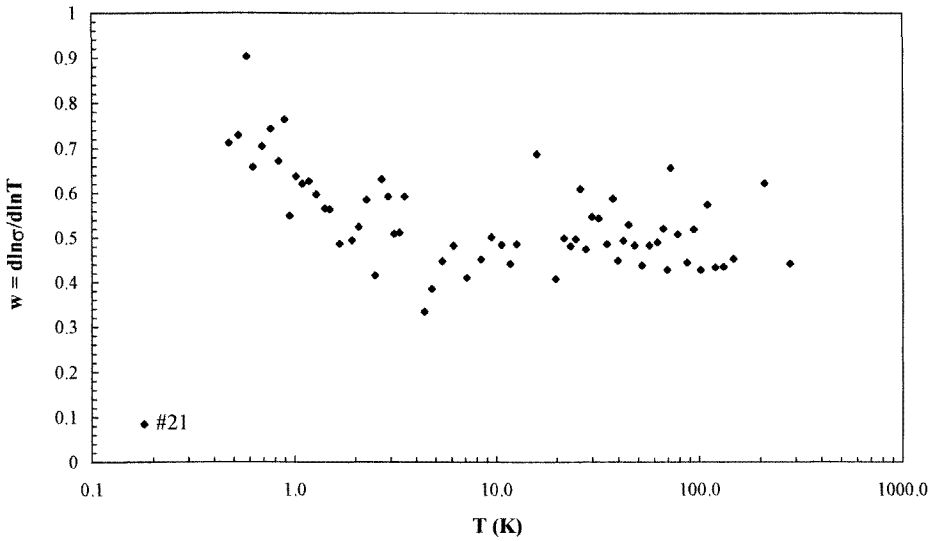

Figure 1. Plot of $w(T) = \mathrm{d}\ln\sigma/\mathrm{d}\ln T$ versus temperature for film No 21, the third film located below the MIT. The *independent temperature* behaviour of $w(T)$ throughout two decades of temperature identifies this film as an insulating film. The average value of $w \approx 0.50$ implies that $\sigma \propto T^{0.50}$ or $\rho \propto T^{-0.50}$.

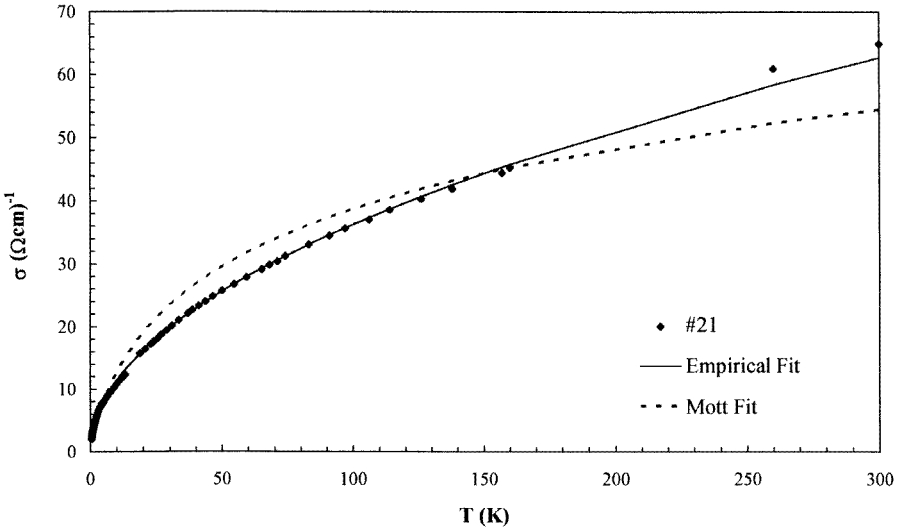

Figure 2. The zero-field conductivity data versus temperature for the weakly insulating film No 21. The full curve is an empirical fit using the expression $\sigma(T) = 3.66T^{0.50}$, which suggests that EEIs dominate the conduction process. The broken curve represents a forced fit of the Mott VRH law through the data. The Mott fit is unacceptable.

of 3 K $< T <$ 300 K, this criterion is not satisfied, and thus the Mott law is not applicable to the conductivity data of this film.

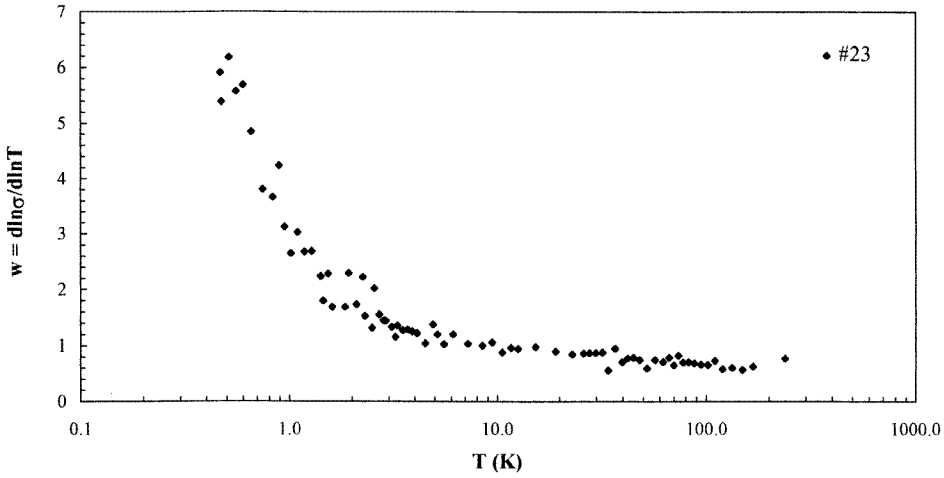

Figure 3. Plot of $w(T) = \mathrm{d}\ln\sigma/\mathrm{d}\ln T$ versus temperature for film No 23, the fifth film below the MIT. The constant w values close to 0.7 are observed only in the limited temperature region of 160 K to 90 K. Below 90 K, the w values slowly increase. Below 10 K this film exhibits activated hopping, according to the rapid increase of the w values.

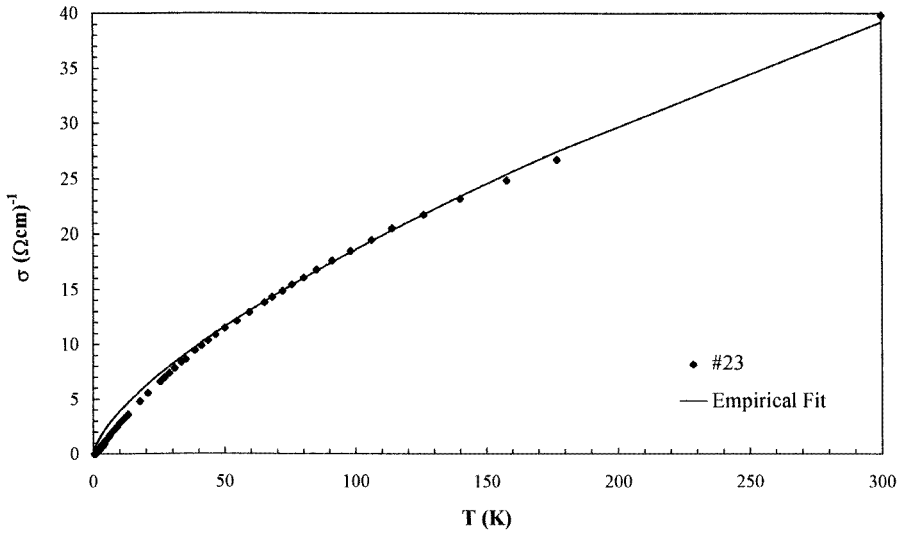

Figure 4. The zero-field conductivity versus temperature for film No 23. Note that the conductivity power-law expression, given by $\sigma(T) = 0.82T^{0.68}$, fits the data only over the limited temperature region of 120 K to 60 K.

Films Nos 22 and 23 contain less nickel than film No 21; their conductivities behave similarly. Film No 23 displays a constant w behaviour in the limited temperature interval of 170 K to 80 K, as seen in figure 3. From 80 K to 3 K, the w values slowly increase, followed by a rapid increase below 3 K. From 300 K to 60 K, the conductivity can be described by the empirical power law $\sigma(T) = 0.82T^{0.68}$ $(\Omega\ \mathrm{cm})^{-1}$, illustrated in figure 4. Below 3 K, the data can be described by an activated hopping expression

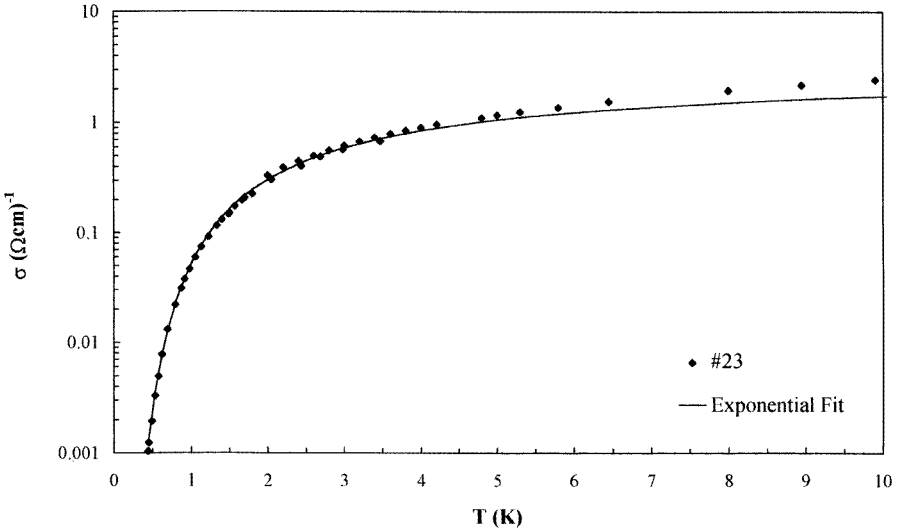

Figure 5. Exponential fit given by $\sigma(T) = 3.43/\exp(6.16/T)^{0.79}$ $(\Omega\ \mathrm{cm})^{-1}$ to the conductivity data of film No 23 in the low-temperature liquid helium regime.

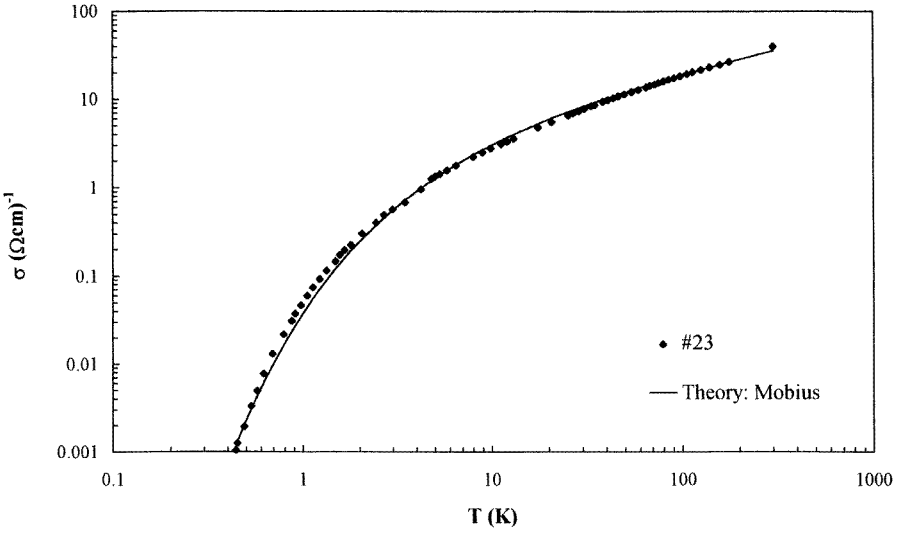

Figure 6. The empirical scaling theory of Möbius compared to the conductivity data of film No 23. Three fitting parameters have been used in obtaining the full curve fit: the prefactor C, the characteristic temperature T_0 and the exponent y.

$\sigma(T) = 3.43/\exp(6.16/T)^{0.79}$ $(\Omega\ \mathrm{cm})^{-1}$ shown in figure 5. The exponent of 0.79 is somewhat close to the exponent of 1, which is characteristic of nearest-neighbour hopping. For samples located just below the MIT, the localization length is expected to diverge as x_c is approached from below the MIT; and nearest-neighbour hopping might dominate with a hopping exponent $y = 1$. It would be useful to extend these data into the millidegree temperature regime to better define the hopping exponent.

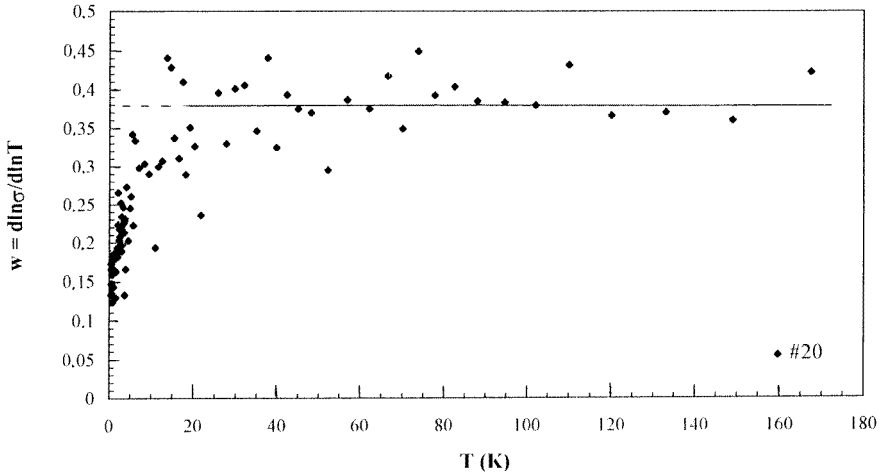

Figure 7. Plot of $w(T) = \mathrm{d}\ln\sigma/\mathrm{d}\ln T$ for film No 20, the second film located slightly below the MIT. Above 25 K, w is temperature independent and is approximately equal to 0.38 implying that $\sigma \propto T^{0.38}$.

Figure 6 illustrates an example of a fit of the Möbius empirical scaling law to the data of film No 23 where the full curve is represented by $\sigma = 2.86T^{1/2}/\exp(3.23/T)^{0.69}$ [14]; recall earlier that the low-temperature $w(T)$ fitting scheme yielded an exponent $y = 0.79$, which is close to this 0.69 exponent. Notice that we have associated the high-temperature conductivity function $\xi_{hi}(T)$ with the high-temperature $T^{0.50}$ results of film No 21 mentioned earlier. Film No 22 can also be fitted equally well with the Möbius expression. Hence, the Möbius scenario suggests that our crossover is from a temperature power-law dependence at high temperatures to a VRH type hopping law at low temperatures and *not* from a Mott hopping law to an Efros-Shklovskii hopping law in these weakly insulating films. This scaling law needs to be confirmed theoretically.

Films Nos 20 and 19, located slightly below the MIT, also exhibit 'crossover regions' but toward metallic behaviour rather than towards VRH behaviour. In figure 7, the w values for film No 19 are temperature independent at 0.38 (definitely less than 1/2) between 180 K to 20 K. Below 20 K, the w values decrease but do *not* extrapolate to zero as $T \to 0$ K, as anticipated for a metallic film. Instead the w values tend to the *finite* value of about 0.14 as seen in figure 8. Clearly this film is insulating since the metallic criterion that $w \to 0$ as $T \to 0$ is not satisfied. If one assumes below 20 K that w decreases linearly with the temperature T going as $w = z + T/T_0$, then *mathematically* the conductivity can be described by $\sigma(T) = CT^z \exp(T/T_0)$. Assuming that T_0 is much greater than T's, the exponential can be expanded to yield $\sigma(T) \approx CT^z + DT^{z+1}$, and the conductivity clearly vanishes as $T \to 0$, making the film insulating. A good fit to the conductivity data below 10 K is given by $\sigma(T) = 12.7T^{0.14}\exp(T/46.9)$ $(\Omega\ \mathrm{cm})^{-1}$. We are not aware of any physical model that would explain this behaviour. Above 20 K, the conductivity can be fitted nicely using the temperature power-law expression $\sigma(T) = 8.61T^{0.38}$ $(\Omega\ \mathrm{cm})^{-1}$ as shown in figure 9. The exponent value of 0.38 is close to the 1/3 prediction of Finkel'shtein [9].

Above 200 K, there appears to be an additional thermally activated conduction process that causes the conductivities to exceed the predictions of the simple T^z power-law process;

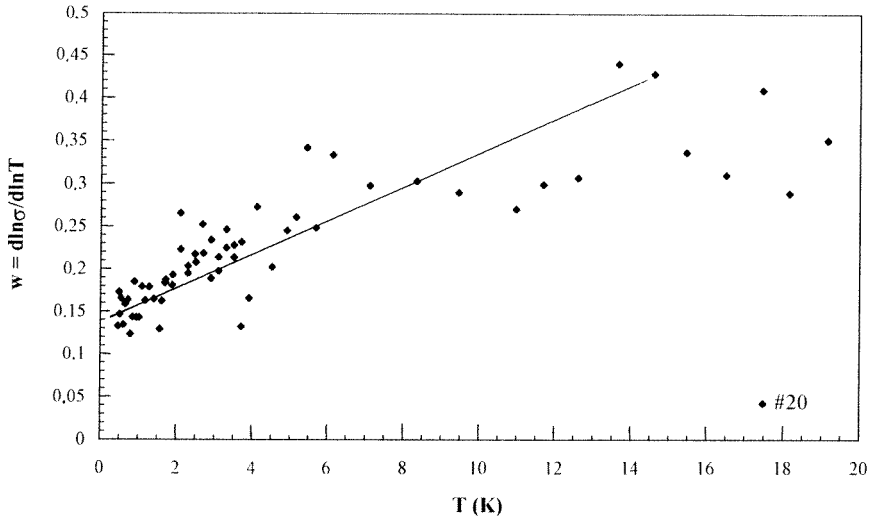

Figure 8. Plot of $w(T) = \mathrm{d}\ln\sigma/\mathrm{d}\ln T$ for film No 20 at low temperatures. Notice that when the w data are extrapolated linearly to $T \to 0$, w takes on the *finite* value of about 0.14 and not *zero*. A metallic film would have $w \to 0$ as $T \to 0$; hence this film is insulating. Refer to the text for a discussion of the temperature dependence of σ below 10 K.

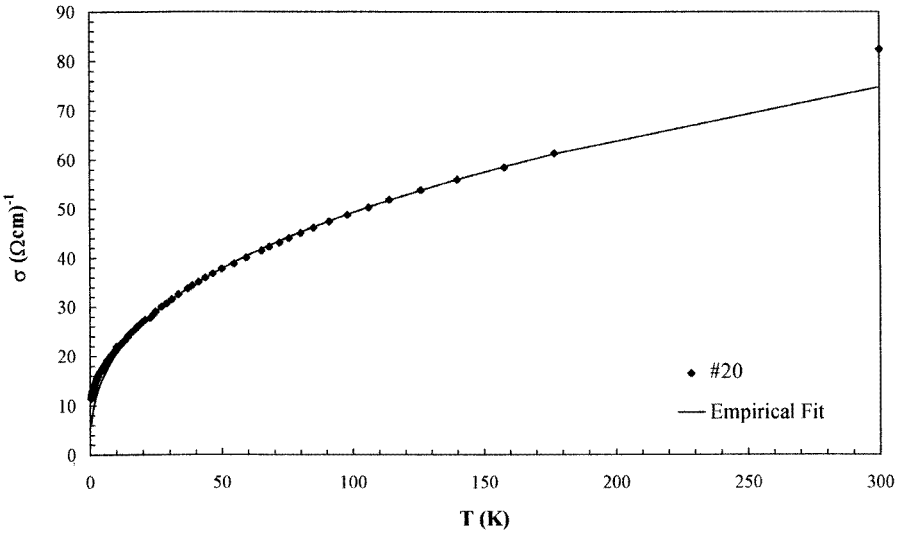

Figure 9. Zero-field conductivity of film No 20, located slightly below the MIT, versus temperature. The full curve is the empirical fit given by $\sigma(T) = 8.61T^{0.38}$ $(\Omega\ \mathrm{cm})^{-1}$; deviations of the data from this fit occur below 15 K.

the differences are noticeable at 300 K in figures 2, 4 and 9. Below 200 K, this contribution appears to be negligible in our films.

Non of the above films fall in the 'strongly insulating' regime. We have fabricated another homogeneous a:Ni_xSi_{1-x} series having much less nickel content. For films having $x \approx 8$ at.% Ni, the conductivity at high temperatures exhibits Mott VRH behaviour [11]

and at low temperatures, ES VRH behaviour [23]. Details can be found in [13] where a useful 3D crossover expression from Mott to ES VRH behaviour is also described. There are now several other 3D and 2D scaling theories that can be fitted to data which display the Mott to ES crossover behaviour [24–26].

5. Magnetoconductance data and comparison to the EEI theory

In the metallic films, the magnetoconductance {MC or $\Delta\sigma = \sigma(B) - \sigma(0)$} data could be well explained using both the EEI theory [15] and the WL theory [16]. On the metallic side, the WL theory made a major contribution of 60% to the negative MC data while the EEI theory gave the remaining 40% negative contribution; see [17] for details. We speculate that immediately below the MIT transition, only the EEI theory contributes to the MC, since the WL theory should break down just above the MIT. Theoretical calculations by Kleinert and Bryksin confirm this speculation [27]. In addition, the Ioffe–Regel condition that $k_F l_e \approx 1$ suggests the breakdown of the WL theory at the MIT since this condition implies that the elastic scattering length cannot be much smaller than the typical lattice spacing [28, 29]. According to Gerd Bergmann's scenario of WL, many hundreds of elastic scattering events must take place before an inelastic scattering event occurs [16]; this multiple elastic scattering process cannot occur if $k_F l_e \approx 1$. Thus, no significant contribution to the magnetoconductance would be anticipated from the WL theory. Measurements on Si:B near the MIT also support this conjecture [30].

Lee and Ramakrishnan [15] have calculated the 3D magnetoconductance (MC) contribution arising from EEI in the particle–hole channel:

$$\Delta\sigma_{EEI}(B,T) = \frac{-e^2}{4\pi^2\hbar}\tilde{F}_\sigma \left(\frac{k_B T}{2\hbar D_{dif}}\right)^{1/2} g_3\left(\frac{g_e\mu_B B}{k_B T}\right) \tag{10}$$

where g_e is the Lande factor, equal to 2.00 for bulk Si [31]. $\tilde{F}_\sigma$ is the electron screening parameter. Ousset *et al* [32] have suggested suitable approximations for the function $g_3(x)$:

$$g_3(x) \approx 5.6464\times10^{-2}x^2 - 1.4759\times10^{-3}x^4 + 4.2747\times10^{-5}x^6 - 1.5351\times10^{-6}x^8 + 6\times10^{-8}x^{10} \qquad x \leqslant 3 \tag{11a}$$

$$g_3(x) \approx 0.64548 + 0.235(x-4) - 7.45\times10^{-4}(x-4)^2 - 2.94\times10^{-3}(x-4)^3 + 6.32\times10^{-4}(x-4)^4 - 5.22\times10^{-5}(x-4)^5 \qquad 3 \leqslant x \leqslant 8 \tag{11b}$$

and

$$g_3(x) \approx x^{1/2} - 1.2942 - \frac{\pi^2}{12x^{3/2}} - \frac{\pi^4}{16x^{7/2}} - \frac{5\pi^6}{32x^{11/2}} \qquad x \geqslant 8. \tag{11c}$$

The limiting forms of g_3 for large and small x are

$$g_3(x \to \infty) \approx \sqrt{x} - 1.29$$

and

$$g_3(x \to 0) \approx 0.0565x^2. \tag{11d}$$

Note that the high-field behaviour of $\Delta\sigma_{EEI}$ has the $B^{1/2}$ dependence observed in the MC data. It is useful to note that $g_e\mu_B/k_B \approx 4/3$ in units of KT^{-1} for $g_e = 2$.

The MC data compared with the EEI theory of equation (10) are shown in figure 10 for film No 20 located just below the MIT. The value of the effective fitting parameter $\tilde{F}_\sigma/D_{dif}^{1/2}$ was found to be equal to 0.93, determined by setting the theoretical predicted value for $\Delta\sigma$ equal to the experimental MC value at $B = 3.45$ T at $T = 0.55$ K. There are no other

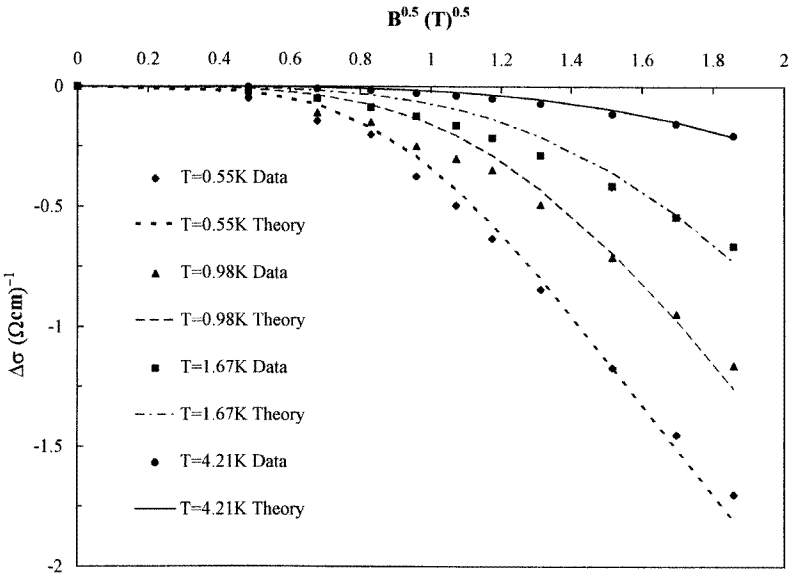

Figure 10. MC data for the insulating film No 20, located slightly below the MIT. The MC contribution from the EEI theory has been used with one fitting parameter. No WL contribution has been included. The agreement reinforces the claim that EEIs play the dominating role in the transport near the MIT in this material.

fitting parameters. Even the fits to the MC at the three higher temperatures in figure 10 are acceptable, suggesting that EEI processes dominate the MC data just below the MIT; no changes in the fitting parameters, $\tilde{F}_\sigma/D_{dif}^{1/2}$ and g_e, were made at these higher temperatures.

6. The 3D EEI theory and the 'weakly insulating' zero field data

EEIs produce a dip in the density of states around E_F in metallic 3D films. This dip results in a small correction to the zero-field 3D metallic conductivity, which reduces the conductivity with decreasing temperatures. According to the 3D prediction of Altshuler and Aronov [33], the particle–hole contribution arising from EEI's to the zero-field conductivity is

$$\sigma_{EEI}(T) = \frac{1.294}{\sqrt{2}}\frac{e^2}{4\pi^2\hbar}\left(\frac{4}{3}-\frac{3}{2}\tilde{F}_\sigma\right)\left(\frac{k_BT}{\hbar D_{dif}}\right)^{1/2} \tag{12}$$

where the electron screening parameter $\tilde{F}_\sigma$ ranges between 0.2 to 0.4 for many thin metallic films. Note the $T^{1/2}$ dependence is in close agreement with the temperature power term determined by using equations (9) and/or (5). Since the empirical fits suggest that $\sigma \approx 3.7T^{1/2}$ $(\Omega\ \mathrm{cm})^{-1}$, one can now solve for $\tilde{F}_s$ and D_{dif} using both the MC and zero-field conductivity data to obtain $\tilde{F}_\sigma \approx 0.32$ (corresponding to 6×10^{20} electrons cm^{-3} if $m^* = m_e$ [34]) and $D_{dif} \approx 0.12$ $\mathrm{cm}^2\ \mathrm{s}^{-1}$ for film No 20. These values should be compared to those of the metallic films above the MIT where $\tilde{F}_\sigma \approx 0.20$ (corresponding to 6×10^{21} electrons cm^{-3} if $m^* = m_e$ [34]) and $D_{dif} \approx 0.25$ $\mathrm{cm}^2\ \mathrm{s}^{-1}$. An increasing $\tilde{F}_\sigma$ value just below the MIT results from a decrease in the free electron carrier concentration n since there is reduced screening and stronger Coulomb repulsion between the electrons [15, 34].

The decrease in the diffusion constant is suggestive of increased electron localization at the nickel impurity sites just below the MIT. In this connection, Anderson introduced a model having a random potential at each potential well site [35]; if the randomness becomes sufficiently great, the electron wavefunctions become localized, resulting in the Anderson MIT. We do not have a clear understanding how the decreasing nickel content influences the disorder and hence the randomness of the potential wells in this amorphous material.

For the strongly insulating films, the EEI expression for the MC given by equation (10) no longer described the MC data, suggesting the break down of this theory far below the MIT.

Although equations (10) and (12) are valid for metallic films, the EEI theory is still expected to be valid in the vicinity of the MIT. According to Finkel'shtein [9], the power-law exponent z is predicted to be somewhat smaller than 1/2, having the magnitude of $1/3 < z < 1/2$ near the MIT. A similar prediction by Newson and Pepper suggests that $z = 1/3$ just above the MIT [10]. The power-law exponent of $z = w \approx 0.38$ observed for film No 20 in figure 7 supports these predictions.

7. Discussion

We speculate on the physics of the weakly insulating regime. Recall that on the metallic side, the conductivity consisted of two contributions according to equation (5): (a) the zero-temperature conductivity term, $\sigma(0)$, that depends upon the metal content x through disorder and (b) the EEI term of approximate magnitude of $4T^{1/2}$ $(\Omega\ \mathrm{cm})^{-1}$ according to equation (12) [15, 33]. At the MIT, $\sigma(0)$ vanishes according to either the scaling theory of Abrahams *et al* [36] or to the Mott concept of a minimum metallic conductivity [37], leaving only the EEI term. Provided that the electrons are not strongly localized, namely that the localization length ξ is large and diverges as the MIT is approached from below, then the EEI process *should* continue to dominate the conductivity just below the MIT. The magnitudes for the C's and z's in the empirical fits of $\sigma(T) \approx CT^z$ certainly suggest this scenario. The exception to the $z \approx 1/2$ value is the large magnitude of $z \approx 2.4$ reported by Friedman *et al* [21] in n-CdSe. Hence, we cannot rule out the possibility that the agreement between these data and the EEI theory is completely fortuitous and that a completely different mechanism could be responsible for the power-law temperature behaviour. Clearly more experimental studies just below the MIT are needed to confirm the EEI scenario.

As the Ni content is further *decreased* making the films *more insulating*, we believe that the EEI mechanism no longer will apply since the electrons are increasingly localized and are no longer able to diffuse. Most likely, VRH starts to dominate the conductivity processes in these more insulating samples. In the most strongly insulating samples, the electrons are strongly localized and VRH between the impurity sites prevails.

If this model is correct, there is still the puzzling question of why a crossover should occur, if at all, from the power law behaviour at high temperatures to activated hopping at liquid helium temperatures. If EEIs are indeed responsible for the high-temperature power-law behaviour, there is no reason why this mechanism should be dominated by an activated hopping process at low temperatures, if these two processes are independent of one another. A possible explanation is based upon a model proposed by Aleshin *et al* [38] and Shlimak [39] to explain the transition in very strongly insulating films from Efros–Shklovskii (ES) VRH with the exponent $y = 1/2$ to simple activated hopping with $y = 1$. These groups suggested that the single particle density of states does not decrease continuously to zero at the Fermi energy as $[E - E_F]^2$ forming the so-called 'Coulomb

gap' [12, 23] but decreases *discontinuously* to zero owing to the presence of a small 'hard' or 'magnetic' gap centred about E_F [38, 39]. The presence of this gap would give rise to the simple activation law with $y = 1$. In analogy, we propose in our scenario that there is a transition from a *square root energy dependence* to either a nearest-neighbour hopping dependence or to a hard gap structure in the density of states. Hence rather than the density of states (DOS) decreasing continuously to zero as $[E - E_F]^{1/2}$ about E_F [40, 41], we propose a crossover in the DOS from an $[E - E_F]^{1/2}$ dependence at high temperatures to an $[E - E_F]^2$ dependence or a stronger power-law dependence including a possible hard gap formation or nearest-neighbour hopping process at low temperatures. This would then explain the transition from a simple power-law dependence at high temperatures towards simply activated conductivity behaviour at very low temperatures. If the VRH conductivity has an exponent y (see equation (4)), then Hamilton [42] and also Pollak [43] have shown that the exponent y is related to the exponent ν in the DOS (DOS $\propto |E - E_F|^{\nu}$) according to the expression $y = (\nu + 1)/(\nu + D + 1)$, where D is the dimensionality of the film. For our case of $y \approx 0.8$, ν would take on the large value of 11.

John Adkins has suggested an alternative and intriguing explanation in place of the nearest-neighbour hopping process [44]. Using a viscous liquid model of strongly correlated electrons (very strong EEIs), Adkins predicts a *simple* activation law for σ. The argument of the activation law contains a characteristic energy $W = k_B T_0$ that represents a pinning energy; this pinning energy depends upon disorder and on the metal content x and is predicted to reduce to zero at the transition to the metallic state [44].

For films located immediately below the MIT, the simple power-law dependence of the conductivity at high temperatures crosses over to a more complicated temperature dependence at liquid helium temperatures with the w's tending to extrapolate to small finite values as $T \to 0$. We currently have no explanation for this behaviour.

The various transport mechanisms which we observe near the MIT are summarized in figure 11.

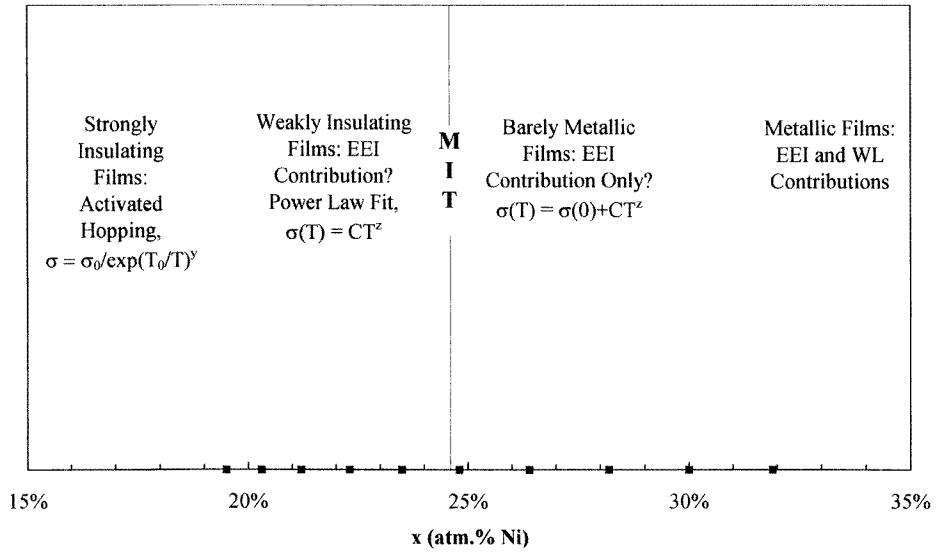

Figure 11. Schematic diagram for the various possible processes that contribute to the zero-field conductivity above and below the MIT. The nickel content scale, x, is based upon EDAX measurements.

Acknowledgments

We are most grateful to Dr Arnulf Möbius of the IFW-Dresden for introducing us to his scaling expression and for fruitful discussions. We acknowledge Dr R Grötzschel of the Rossendorf Research Center of Dresden for the RBS results and to Dr H D Bauer of the IFW-Dresden for additional EDAX results. We are indebted to Dr C J Adkins of the University of Cambridge and Dr A Palevski of TAU for informative discussions and to Dr A Gladkikh of TAU for film characterization. We thank Drs M Ahlgren and M Rodmar of the Swedish Royal Institute of Technology for reviewing the paper and Mrs Rachel Rosenbaum for the editing. We greatly acknowledge the German-Israeli Foundation (GIF) for its warm financial support. The Electron Microscope Unit thanks the University of the Witwatersrand for financial support via the Microstructural Studies Research Programme.

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